Socrates and Hume at Billiards

Socrates: You wanna go double or nothing?

Hume: Well, okay, I guess. Are you sure you never played billiards before? I thought you said it hadn't been invented yet in ancient Greece.

S: I might have played a little bit since then. You rack 'em. I want to read this comment that just appeared on the internet. This guy Julian is commenting on Aguilar's paper, *Did Mises and Hayek Predict the Great Depression?*

H: What does he say?

S: He says, "I'm having a hard time with some of the things you say, this ['If the predicted phenomena were observable, one would just observe them and forget about theory.'] being an example. I don't suppose ANY theory worth its salt would predict Unobservable phenomena??"

H: Hmmm. What do you think?

S: All theories worth their salt make predictions about phenomena that will *eventually* be observable. It's pointless to make predictions about things that cannot be observed.

H: For instance?

S: For instance, I might ask, "How many angels can sit on the head of a pin?" One theologian says none, because an angel is as big as a person – she would just poke herself in the derriere if she tried to sit on a pin. Another theologian says a million and a third says it could be an infinite number.

Clearly, none of these theories are worth their salt because we cannot now and never will be able to observe even one angel, much less a million of them, sitting on the head of a pin or anywhere else.

H: But then aren't you conceding the commentator's point?

S: No. That is not what Aguilar is talking about in his paper. Theories make predictions about future events that are not *yet* observable. The events *must* be in the future, because, if the predicted phenomena were observable *now*, one would just observe them and forget about theory. The question that divides us is, without knowing yet how a theory will perform on the question at hand, how do we decide which theory to use?

H: Give me an example.

S: Suppose I ask, "If I hit the cue ball into the object ball on this billiard table at this angle [indicates with his cue stick], where will the object ball go?" Clearly, the predicted phenomena will be observable *after* I make the shot, but then it's too late for me to correct my aim. So it is that I put out the call *now* for theorists who claim they can tell me, before I make the shot, where the ball will go.

I get two responses:

A) An old man with a flowing white beard wearing a black robe with silvery little stars and moons printed on it shuffles into the pool hall. He tosses some tea leaves onto the table, studies them for a moment and then announces, "In the corner pocket." Reliable witnesses step forward to tell me that our swami has made similar predictions in the past and has always called the shot correctly.

B) A scientist in a white lab coat announces, "Two of the principle axioms of physics are that momentum and energy are both conserved. If you measure the momentum (mass times velocity) and the energy (mass times velocity squared) of the cue ball before the collision and compare these numbers to the sums of the cue and object balls' momentum and energy after the collision, then, within this axiomatic system, they must be equal. Thus, you can answer your question by solving two simultaneous equations in two unknowns."

"Okay," I say, turning to the swami, "I've got two questions. First, is it really true that you have never been wrong?"

"Absolutely!" he says.

"And, second, did you bring a vacuum cleaner?"

"Ha! Ha! Ha!" he laughs and shuffles out the door.

"Okay," I say to the scientist, "I think I understand what you just said, but which pocket is the object ball going to drop into?"

"I don't know. I've never played this game before and I have no means of measuring things like rotational energy or the heat energy produced by the friction of the ball rolling across the felt. You asked me what the science of physics could contribute to your debate and I told you. Now where's my beer?"

"Hmmm," I muse as I fetch the physicist his beer ("Will Solve Physics Problems for Beer," his ad had read), "it's a dilemma. Perhaps we should consult the stars."

H: You mean astrology?

S: No, the stars of economic theory: Milton Friedman.

"Milton Friedman, torchbearer for the 'free market', insisted that the realism of background assumptions is not important. 'In general, the more significant the theory, the more unrealistic the assumptions'. Good theory may well make use of 'wildly inaccurate' assumptions, and proceed 'as if' the assumptions held true. The purpose of theory is to generate testable implications." quoth Cristobal Young.

H: Friedman sounds like he supports position (A). The swami's assumptions, that tea leaves have anything to say about billiards, can only be described as unrealistic, yet his implication, that the ball will drop into the corner pocket, is definitely testable. He claims, and I believe him, that similar implications of his have been tested in the past.

S: Okay, but what does that other star of economic theory, Victor Aguilar, have to say on the subject?

H: Who?

S: You know, the guy who wrote the famous book, *Axiomatic Theory of Economics*, in 1999.

H: What book?

S: Okay, so maybe he's not the star that Friedman is, but let's read what he has to say anyway (ATE, pp.98-99):

"Only a few hundred years ago it was common to segregate assertions into relations of ideas, such as the Pythagorean Theorem, and matters of fact. An example of the latter was given by David Hume in l748:

> When I see, for instance, a billiard ball moving in a straight line toward another,... may I not conceive that a hundred different events might as well follow from that cause?... All these suppositions are consistent and conceivable. Why then should we give preference to one which is no more consistent or conceivable than the rest?

"However, the term "consistent" is only meaningful in reference to a specific set of axioms, in this case those of Newtonian Mechanics. Only one of the hundreds of different events mentioned in the above quotation conserve both momentum and energy, so, in reference to these two axioms, all the other events are inconsistent. The purpose of having a theory at all is that one does not have to apply experience to every event but need only apply it once when deciding on one's axioms, in this case that momentum and energy are both conserved. After that, application of the theory is algorithmic. The only reason that the assertion of how billiard balls will react to a collision was considered a matter of fact and the Pythagorean Theorem was considered a relation of ideas is because Euclid's five postulates were older then Newton's laws of physics at the time. As it turned out, both sets of axioms were shown to be only one of several possible systems, each intuitive in its own way and each internally consistent. Economics is younger now than physics was in 1748 and no one of Newton's stature has been attracted to it. But nothing prevents finding a set of axioms for economics just as Newton found a set of axioms for physics. This is what I propose to do in **Axiomatic Theory of Economics**."

H: Hey! He's quoting me! It sure is cool to think that people are still reading my book, *On Human Nature and the Understanding*.

S: Yes. Yes. But how does Aguilar's statement relate to our current question?

H: Aguilar sounds like he supports position (B). The physicist's assumptions, that momentum and energy are both conserved, are axiomatic, yet his implications require equipment for measuring things like rotation and frictional heat that we do not possess.

S: Maybe we can approximate. If we assume that the balls float on a cushion of air, then we don't have to consider rotation or friction. The resulting implications may not achieve the accuracy that the theory is capable of, but they should be close enough to tell us which pocket the ball will fall into.

H: I'm a little fuzzy on what the physicist meant when he said, "you can answer your question by solving two simultaneous equations in two unknowns."

S: On a two-dimensional table, the unknowns are vectors, which make the equations complicated, so I'll demonstrate in one dimension.

[Socrates hits the cue ball directly into the object ball, leaving the cue ball motionless and the object ball moving directly away from it with the same velocity that the cue ball had before the collision.]

Let v_c be the cue ball's velocity and v_o be the object ball's velocity. Before the collision, $v_c = 10$ cm/sec and $v_o = 0$ cm/sec. So adding up the total momentum and the total energy defines two equations:

$$v_c + v_o = 10 \qquad \text{momentum}$$
$$v_c^2 + v_o^2 = 100 \qquad \text{energy}$$

H: Wait a minute! What happened to mass?

S: Both balls have the same mass and we can ignore a constant multiple on both sides of an equation. If they had different masses, like a bullet hitting a block of wood, we'd have to keep track of mass.

H: You mean like in a ballistic pendulum? That's how armorers measured the speed of bullets back in my day, before they invented electronic chronographs.

S: Yes. Ballistic pendulums would be a good topic for further research on this subject. But, right now, do you see how to solve the two equations for v_c and v_o?

H: Okay, I remember this from algebra. We solve both equations for v_o and then set them equal to each other. Then, squaring both sides, we get $(10 – v_c)^2 = 100 – v_c^2$.

S: Right. Now, moving everything to one side, we get $2 v_c (v_c - 10) = 0$. So there are two possible solutions: $v_c = 0$ or $v_c = 10$. Using our first equation, $v_o = 10 - v_c$, we have $v_o = 10$ or $v_o = 0$. We already know that $v_c = 10$ and $v_o = 0$ describes the situation before the collision, so $v_c = 0$ and $v_o = 10$ must describe the situation after the collision.

H: So I was wrong in my book, *On Human Nature and the Understanding*, when I said that "a hundred different events might as well follow from that [collision]… All these suppositions are consistent and conceivable." Dang it!

S: "Conceivable" maybe, but the term "consistent" is only meaningful in reference to a specific set of axioms, in this case that momentum and energy are both conserved.

H: Okay, but why aren't you convinced by the swami's unfailing history of accurate predictions?

S: I believe that he has never been wrong, but I'm not convinced that his history implies that he will get today's prediction right. As Aguilar said (ATE, pp. 33-34), "in mainstream logic any conclusion can be attached to a false premise and the implication is still considered to be true… The new system [of formal logic] introduced here resolves this problem by making the statement 'all p are q' synonymous with 'if p then q,' a route that mainstream logic could not take, as it separates truth-functional statements (of which 'if p then q' is one) from quantificational statements (of which 'all p are q' is one)."

H: What?

S: Even if this is the swami's first such prediction ever, by the standards of mainstream logic, he is not lying if he affirms the question, "Is it really true that you have never been wrong?"

H: I think I see what you mean.

S: And, even if we grant that the swami has made similar predictions in the past, how many were there? Two? Two hundred? And what does "similar" mean anyway? Were his previous predictions also about bank shots? Or just straight-in shots? Maybe his magical powers only work on green felt tables and he'll fall flat on his face predicting events on our beige table.

We have no way of knowing what he means by a similar prediction.

H: You're right. We had better use the physicist's theory. Just because he has never played billiards does not mean that he has nothing to say on the subject. The assumption that momentum and energy are always conserved has proven sound for hundreds of years in everything from planetary motion to apples falling on my friend Isaac Newton's head.

S: Name dropper! You didn't even know Newton.

H: I did too!

S: If you say so. But you're right that we should use the physicist's theory. As Aguilar said (ATE, p. 98), "One tests phenomena for conformance to one's axioms and then one assumes that phenomena conform to the implications of those axioms. One does not arbitrarily assume that one's axioms are true and then test phenomena for conformance to the implications of those axioms."

H: Told you so.

S: Told me so, huh? Okay, smarty-pants, then give me another example of an axiomatic system.

H: Well, um, let me think. Gee, physics was never really my strong suit.

S: Think economics. It's something that everybody learns in Econ 101.

H: All I remember from Econ 101 is the Keynesian Cross. The professor didn't say *anything* about any axiomatic systems.

S: The identity, $Y = C + I + G + X$, national income equals consumption plus investment plus government spending plus net exports, *is* an axiom. How else would you describe such a statement?

H: So, when economists define consumption to be a constant multiple of national income, they have one equation in one unknown, Y.

S: Right. That constant is called the marginal propensity to consume, usually about 0.9. That means that people typically spend about 90% of their income on consumption goods.

H: Unless they're Americans – then they spend 110%. Ha! Ha!

S: Groan. I think we're supposed to assume it's 90%. Here, let me show you…

H: Wait! I can do this. Don't help me. By substituting $0.9\,Y$ in for C we get:

$$Y = 10\,(I + G + X)$$

Net exports, X, are determined by conditions in the rest of the world and can be ignored for a big country like the United States. Investment, I, is determined by "animal spirits," that is, investment bounces up and down erratically because investors are assumed to be a bunch of irrational morons. When investment happens to bounce downwards it would cause a depression unless the government steps in with big public works projects like the WPA, which built the national parks in the 1930s.

S: Very good! Keynes' basic thesis was that investors are irrational and the government must rescue capitalism from the capitalists with public works projects.

H: So, it sounds like mainstream macroeconomics is easier than Newtonian Mechanics because they only have one equation while the physicists have two.

S: Actually, one equation is inadequate – you get absurdities like the assertion that increasing government spending always produces a ten-fold increase in national income. When too many students started to complain about the absurdities, John Hicks expanded on Keynes' theory by adding another axiom.

Today, students satisfying their social science requirement by taking Econ 101 only hear about the first equation, $Y = C + I + G + X$. However, economics majors who go on to take an upper-division macroeconomics course will be given another equation, $D = M/P$, the demand for money equals the stock of money divided by the price level, and another variable, R, the interest rate.

H: I thought upper-division macroeconomics courses were about IS-LM Analysis.

S: They are. IS-LM Analysis is a system of two equations in two unknowns, just like the physical system we just discussed, except that the equations are $Y = C + I + G + X$ and $D = M/P$ with unknown quantities, Y, national income, and R, the interest rate.

H: I don't see "R" in that equation.

S: "The demand for real money holdings is positively related to income because one needs to hold currency for the day-to-day transactions one makes, and the higher one's income, the more transactions one makes every day. Mainstream economics asserts that the demand for real money holdings is also negatively related to the interest rate because money that one is holding in currency is not earning any interest and, if the interest rate is high, one conserves on one's money holdings to put more in the bank (ATE, p. 243)."

However, this has become controversial because, as Aguilar notes (ATE, p. 243), "economists increasingly assert that most people hold as little of their wealth in money as is convenient regardless of the interest rate." Thus, while the left side of the second equation, D, is positively related to Y and negatively related to R, its exact form depends on which mainstream economist one talks to. For the purposes of our discussion, I will just say "D for demand" and omit the details.

H: How does Aguilar define D?

S: Aguilar is not a mainstream economist. He has his own axiomatic system. That's why this material about IS-LM Analysis is in an appendix (ATE, Appendix C, pp. 231-269), not the main body of his book.

H: So mainstream macroeconomics is just as hard as Newtonian Mechanics?

S: Harder. Mainstream macroeconomics actually has a third equation, $P = \prod(A)$, and a third unknown quantity, P, the price level.

H: I definitely don't remember seeing that equation, not even in grad school.

S: And you won't see it there either. The only person I know of who thinks of mainstream macroeconomics as an axiomatic system with three equations in three unknowns is Victor Aguilar.

"All of the functions in this model are of seven variables: Y, R, P, g, m, r, and t with t representing time. Denote the unknown variables by $\mathbf{y} = [\ Y\ ,\ R\ ,\ P\ ]$ and the known ones by $\mathbf{x} = [\ g\ ,\ m\ ,\ r\ ,\ t\ ]$. **y** is the vector of variables for which the model is solved and **x** is the vector of variables which are taken from historical data… As noted in the previous section, $\varphi(\mathbf{x},\mathbf{y})$ has three coordinate functions, each of seven variables… Consider the function $\mathbf{y} = \psi(\mathbf{x})$, which returns the **y** such that $\varphi(\mathbf{x},\mathbf{y}) = 0$ for a given value of **x**. $\mathbf{y} = \psi(\mathbf{x})$ defines a surface in seven-dimensional space, but not necessarily one that is known explicitly… The derivative of $\psi(\mathbf{x})$ is written $\psi_{\mathbf{x}}(\mathbf{x},\mathbf{y})$. Both **x** and **y** may appear in the functional notation of ψ and $\psi_{\mathbf{x}}$… $\psi_{\mathbf{x}}(\mathbf{x},\mathbf{y})$, the slope of the vector tangent to the surface of solutions to $\varphi(\mathbf{x},\mathbf{y}) = 0$ whose projection onto **x** space represents changes in g, m, r, and t, could be called the multiplier (ATE, p. 233, 253, 255, 256)."

H: I remember the word "multiplier" from Econ 101. But a surface of solutions in seven-dimensional space? That's too tough for me. I know how to find the point where two lines cross on a flat plane, but that's where it stops.

S: Actually, the beauty of mathematics is that once a technique has been developed for a simple two-dimensional case that can be illustrated on graph paper, the technique can easily be extended to more dimensions. Just because you cannot sketch what you are doing on paper does not mean that the problem is any more difficult than the previous one that was accompanied by an illustration.

H: If you say so. Do you think I should read Aguilar's book even if I don't have a degree in mathematics?

S: I think *everybody* should read Aguilar's book. The linear algebra is all in an appendix. To understand the proofs in the main body of the book only requires a basic understanding of multivariable calculus. People who don't have that can just skip over the proofs. They are clearly marked and, since the theorems are stated in plain English, reading the book sans proofs results in no real loss of continuity.

Now, c'mon, we were going to go double or nothing on this next game, right?

H: Right. I get to break. You watch. I'm going to run the table this time. You won't even get to shoot.

S: We'll see.