# Review of Fuerle's *Pure Logic of Choice*

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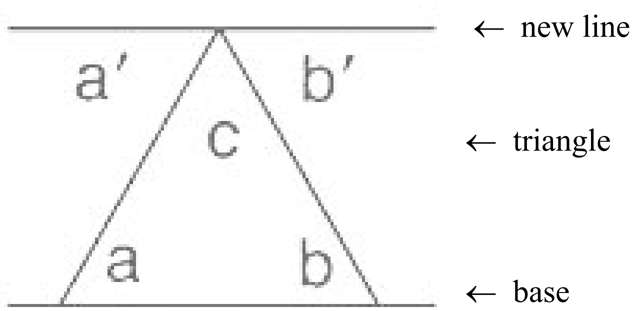
In 1986 Richard Fuerle published a book, *The Pure Logic of Choice*, which "was welcomed by the Austrian establishment (Rothbard, Kirzner, etc.) like a scorpion in a sleeping bag." Ironically, 1986 was when I was working on the early chapters of my own book, *Axiomatic Theory of Economics*, including Chapter Two, "Epistemology." If Rothbard had not been such a coward and had just reviewed Fuerle's work instead of blacklisting him (as he would blacklist me a decade later), I would have read the review and bought the book. Instead, it waited until now, 2006, for a philosophy student to contact me and ask for my opinion on Fuerle's book.

While *The Pure Logic of Choice* is twenty years old now and Fuerle has, in the meantime, retired, it is never too late for a review. Hopefully, we will not have to wait until the year 2019 for someone to defy Rothbard's blacklisting of me and review *Axiomatic Theory of Economics*. But, even if I am retired by then, I will still welcome a review, as Fuerle welcomed hearing from me.

In his chapter on positivism, Richard Fuerle writes:

> A deductionist would verify the geometrical law that all triangles have 180° (i.e., if any angle of a triangle changes, one or both of the other angles must also change so that they sum to 180°) by placing a line parallel to one of the sides of any triangle at the apex of the other two sides.




> Since the internal opposite angles formed when a line crosses parallel

> lines are equal (a' = a and b' = b), and a straight line is defined to be 180°, it necessarily follows that the three angles of the triangle, a, b, and c, must total 180°. The positivist, however, would attempt to verify that geometric law by measuring the angles of thousands of different triangles. He might arrive at the conclusion that the angles total 180° within a probable error of ± 0.01 percent, but he could never prove that the answer was exactly 180°, and he could never prove that the next triangle he measured must total 180° and not an entirely different number (p. 53).

I agree with Fuerle's condemnation of positivism. Elsewhere on this website, I have written a paper, *Socrates and Hume at Billiards*, comparing how a positivist and a physicist would answer a question about where a billiard ball was going to go after being struck by the cue ball. I have Socrates and Hume decide in favor of the physicist, not the positivist. However, as should be clear from reading *Socrates and Hume at Billiards*, I do not agree with Fuerle when he writes:

> I can not be certain that every physical law holds true everywhere and at all times, but, given necessary conditions, I am certain that the laws of economics, like the laws of geometry, do. It is the view that economics is an empirical science like physics that is responsible for the disarray and error that is so widespread in the discipline, and the quite justified contempt that members of the general public have for economics (p. xv).

I will not deny that disarray and error are widespread in economics, but I will deny that physics is an empirical science. Geometry, physics and economics are all axiomatic. How could such important fields of inquiry differ on so basic an issue? But this does not mean that their laws hold true everywhere and at all times. An axiom is a proposition that is assumed without proof for the sake of studying the consequences that follow from it. Whether one's axioms are wisely chosen in the sense that they have any application to the real world is an entirely separate question from whether one's theorems actually follow from one's axioms.

The two axioms discussed in my paper, *Socrates and Hume at Billiards*, are that momentum and energy are both conserved. I think everyone will agree that these axioms were wisely chosen. Neither axiom has let us down in the hundreds of years since Newton founded modern physics. The three axioms discussed in my book, *Axiomatic Theory of Economics*, are that value conforms to the definitions of total ordering, marginal utility and proportionate effect. I think that these axioms are also wisely chosen, though others may disagree. People may, though nobody has yet, also find fault with the proofs of my theorems, feeling that my theorems do not actually follow from my axioms. Hopefully, it will eventually be agreed that my axioms, like Newton's axioms, are wisely chosen and *also* that my theorems are soundly proven. Right now, however, the only point I wish to make is that these are two separate questions.

While Fuerle's purpose in writing the passage quoted above is to condemn positivism, it is actually the first part of this passage, "a deductionist would verify the geometrical law that all triangles have 180° by placing a line parallel to one of the sides of any triangle at the apex of the other two sides," that I take issue with.

Recall that the term "deduct" means to remove something. Yet the first thing Fuerle's deductionist did was to add something. He added the new line at the top of the figure which touches the apex and is parallel to the base. How can this process be called deduction if it requires adding something new? Yet the result, that the interior angles of a triangle sum to 180°, refers only to the original figure, the triangle, and makes no mention of the new line. So, apparently, this new line was added and *then* deducted, though Fuerle does not explicitly say so.

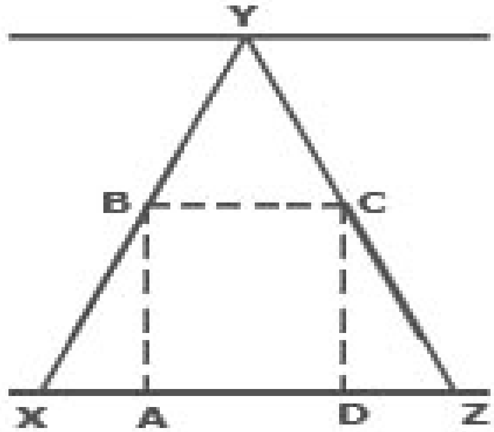
In fact, this is an example of what I refer to as synthetic *a priori* knowledge. In my book, *Axiomatic Theory of Economics*, I write:

"While there are a finite number of theories implied by a given one, there is an infinite number of theories that could have implied a given theory. These theories are found by adding definitions (which are always available) to an alternative of the given theory. If, out of this infinity of theories, the analysis of one of them yields an alternative that contains only the characteristics of the given theory (which is implied by the one under analysis) and yet imply a relation that is not contained in that given theory, this relation is synthetic *a priori* knowledge...

"The use of additional definitions which are then deducted after a solution has been found is often forgotten, leading people to believe that synthetic *a priori* knowledge is impossible and that all understanding is analytic. That synthesis is a passing event which leaves no mark on its creation and that all declarative sentences are analyzable from discursive postulates has led many linguists to take this stand...

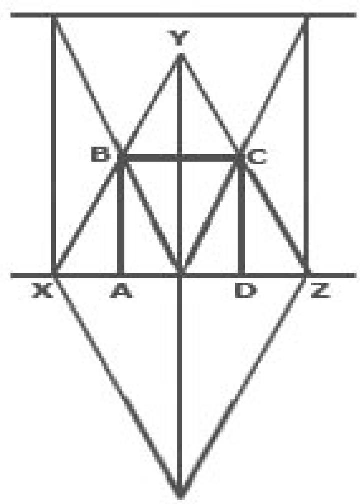
“An illustration of synthetic *a priori* knowledge will now be given:

“If definition *p* is of a given equilateral triangle XYZ and definition *q* is of a square, what additional definition(s) *r* must be added to *pq* so that the analysis of *pqr* leaves only the characteristics of *p* and *q* (there are no superfluous lines) but with the square given definite size, ABCD, so that it fits inside triangle XYZ, as shown in figure 6?




“Neither the definition of *p* (three equal sides of definite length) nor the definition of *q* (four equal sides of indeterminate length) contains any information about the position of B or C. Additional characteristics are needed (lines have to be drawn) to find these lengths. But these additional characteristics have to be deducted again for the new theory to keep the same extension...

“Figure 7 illustrates the solution. This figure, *pqr*, is an anti-implication of *pq*, as it can imply *pq* by deducting the additional lines just added. They are not needed for phenomena to conform to the square inside the triangle. After their deduction, however, the relation of B and C to lines XY and YZ, which was not known before, is still there. This relation is synthetic *a priori* knowledge.



"A more algebraic example is the integration of $\frac{1}{\ln(x)}$. The first three steps establish the needed anti-implication.

$$\int \left[\frac{1}{x\ln(x)} + \frac{x}{x\ln(x)} - \frac{1}{x\ln(x)}\right] dx$$

Multiply by $\frac{x}{x}$, add and subtract $\frac{1}{x\ln(x)}$.

$$\int \left[\frac{1}{x\ln(x)} + \frac{e^{\ln(x)}-1}{x\ln(x)}\right] dx$$

Substitute $e^{\ln(x)}$ for $x$ in numerator.

$$\int \left[\frac{1}{x\ln(x)} + \frac{1}{x\ln(x)} \sum_{n=1}^{\infty} \frac{\ln^{n}(x)}{n!}\right] dx$$

Substitute for the numerator its Taylor series expansion.

$$\int \frac{dx}{x\ln(x)} + \int \sum_{n=1}^{\infty} \frac{\ln^{n-1}(x)}{xn!}\, dx$$

Separate and bring $\frac{1}{x\ln(x)}$ into the summation.

$$\int \frac{du}{u} + \int \sum_{n=1}^{\infty} \frac{u^{n-1}}{n!}\, du$$

Substitute $u = \ln(x)$ and $du = \frac{dx}{x}$.

$$\ln(\ln(x)) + \sum_{n=1}^{\infty} \frac{\ln^{n}(x)}{nn!} + c$$

Anti-differentiate and put $\ln(x)$ back.

"Mathematicians will confirm that much of what they do involves finding a 'trick', usually an identity, which transforms a given equation into one which superficially seems more complicated but which in fact easily implies what they are trying to prove. Almost invariably, once this trick is discovered, the rest of a proof is just a matter of algebraic manipulation (pp. 43-48)."

Fuerle does not acknowledge the possibility of synthetic *a priori* knowledge in his book, *The Pure Logic of Choice*. Of course, neither do any other Kantian philosophers, so it would be unfair to single Fuerle out for special condemnation. Nevertheless, this is the principle weakness of his epistemology.

I would, however, like to commend Fuerle for making an effort to establish a foundation for his science before plunging directly into applications, which is the more common approach. He writes (p. xvi), "When Alice asked the Queen of Hearts where to begin, she was told to begin at the beginning. The beginning is epistemology – the study of how to validate and verify knowledge – and ontology – the study of the nature of those things about which we wish to acquire knowledge. Most [economists] have not begun at the beginning." Fuerle did begin at the beginning and kept at it for a good 80 pages, which is commendable, if a bit off-putting.

Furthermore, Fuerle writes (p. xvii), "While von Mises must be given credit for generalizing the subjective, deductive approach to economics into an overall theory of human action, von Mises rejected the idea of presenting his theory in the form of universal axioms, preferring instead to begin with the common experience of all mankind." I agree. Mises spent a lot of time talking *about* deduction, but very little time actually *doing* deduction. His "action axiom" is really just a platitude. As I ask in my *Critique of Austrian Economics*, "Whoever heard of an axiomatic system with only one axiom? There are only postulate *sets* (e.g. Euclid has five, Kolmogorov has five and this author has three) (p. 34)."

Unfortunately, Fuerle confuses his conception of epistemology – the study of how to validate and verify knowledge – with the invention of new theories. He takes what I refer to in my book as the linguist's approach:

> As linguists deal with theories whose creation has been forgotten and which have turned into statements that could have as easily been handed down from a mountain as synthesized, it is not surprising that they should regard these as cases of analytic knowledge. They need only ask "What do the words mean in this configuration?" and they know the meaning of

> the theory. They forget that at one time the theory was unknown but a simpler one was known without certain relations. Then people noticed that, whenever they used the theory, the phenomena that conformed to it had those relations, but they were hesitant to risk anything on the assumption that future phenomena would have those relations also, for they could not be sure that it was not a coincidence. Then someone found an anti-implication of the theory which, when analyzed, yielded those relations as synthetic *a priori* knowledge, so people no longer had to wonder if the next phenomenon that conformed to their theory would have those relations but could relax and say "Whatever phenomena conform to these characteristics has these relations." Or it might have happened another way and it was some of the characteristics which people were unsure of and someone found an anti-implication of the known relations that, when analyzed, yielded those characteristics as synthetic *a priori* knowledge. But that has all been forgotten and now linguists only see a theory with certain characteristics and certain relations, so they analyze it and then proclaim that synthetic *a priori* knowledge is impossible (pp. 44-45).

The difference between the linguist and the positivist is in their method, not their approach. Both the linguists and the positivists are dealing “with theories whose creation has been forgotten and which have turned into statements that could have as easily been handed down from a mountain as synthesized” and are interested in validating and verifying them. The difference is in how they go about verification of the theories they are given.

The linguist analyzes the meaning of the words and symbols in the result. For instance, if given the statement

$$\int \frac{1}{\ln(x)}\,dx = \ln(\ln(x)) + \sum_{n=1}^{\infty} \frac{\ln^n(x)}{nn!} + c$$

he would inquire about the meaning of the symbols ln (natural logarithm), $\infty$ (infinity), ! (factorial) and $\Sigma$ (summation) in order to know what the formula means. The positivist would insist on evaluating the integral to determine the area from, say, one to ten, and then comparing the result to the output of a numerical integration program which divides the area under the graph into a number of little rectangles and then adds them up.

Clearly, the positivist's approach has weaknesses. He can divide the area under the graph into a couple dozen or even a hundred little rectangles, but he cannot divide it into an infinity of rectangles. Neither can he evaluate more than the first dozen or so terms of the infinite summation. So there will always be some uncertainty in his conclusion. Not only that, but how can he be sure that the accuracy he measures when integrating from one to ten is comparable to the accuracy that will be realized by someone who needs to know the area under the graph from, say, five to twenty?

The linguist's approach is relevant because, if one does not know what the symbols represent, one has no way of evaluating the formula. But neither the linguist nor the positivist have answered the most important question of all: *If one is presented with* $\int \frac{1}{\ln(x)} dx$ *on a calculus test, how is one supposed to respond?*

The professor is not going to accept a computer printout of a numerical integration program and telling him what natural logarithms are is not going to win many points. The bottom line is, how is one supposed to figure out how to integrate $\frac{1}{\ln(x)}$?

It is this and similar questions that I hope to answer, at least in a general way, with my discussion of how synthetic *a priori* knowledge is possible.

## NIHILISM

In his 1986 book, Fuerle includes a chapter titled "Free Will" in which he asserts, basically, that people have free will. I see no evidence that, in 1986, he felt that the existence of free will precluded the use of mathematics in economics. Except for occasionally invoking geometric proofs (such as the one about the interior angles of a triangle summing to 180°) as an ideal for economic theory to emulate, *The Pure Logic of Choice* is silent on the subject of mathematics in economics.

In a recent e-mail, Richard Fuerle has expanded on his views of free will, specifically stating his opposition to the use of mathematics in economics.

> As you know, the Austrians argue that mathematics cannot be applied to economics because economics deals with entities, e.g., prices and quantities bought and sold, that can change arbitrarily and unpredictably. Mathematical modeling requires the world that is modeled to behave rationally and predictably… But values, which determine prices, can jump all over the place, even though the usually don't. That is, there is no underlying law that determines the value that people place on things… There is no underlying law because people have free will. An attempt to apply mathematics to values, i.e., to epistemically correlate a mathematical model with people's economic decisions, will be highly approximate, especially at times of stress.

That is quite an indictment. My theory fails during times of stress? I had better be careful not to let myself get stressed out about this e-mail or I might see my entire theory collapse around me.

Actually, if prices and quantities bought and sold were really arbitrary and unpredictable, that would preclude application of *any* economic theory, regardless of how much or how little math it employed. All economic theories, from *Human Action*, which eschewed all mathematics, to the latest article in the *Journal of Economic Theory*, claim to make meaningful statements about prices. If prices are really arbitrary and unpredictable, then a lot of people have been wasting their time writing about them.

Part IV of Fuerle's book describes many "laws" of economics and, specifically, of prices. This is difficult to reconcile with his later claim that prices and quantities bought and sold are arbitrary and unpredictable. For instance, Fuerle writes:

> Prices are the bits of knowledge that influence individuals to coordinate their plans. To the extent that the conditions of the Law of Quantity Demanded apply, a higher price for good A will influence individuals to alter their plans by foregoing purchasing good A, and perhaps instead purchasing good B. Under these same conditions, to the extent that the conditions that the Law of Supply and Demand apply, sellers will ask a higher price for good B, which will influence other individuals to alter their plans by foregoing purchasing good B and instead perhaps purchasing good C, and so on. Like dropping a pebble into a pond, the effect ripples through the economy, each person voluntarily altering his plans so as to coordinate them with the plans of others. In this way, a "spontaneous order" arises, which was the result of purposeful behavior, yet which no single mind conceived and planned. This is the miracle of the free market (p. 132).

Miraculous indeed if prices are actually arbitrary and unpredictable! Or perhaps prices are only arbitrary and unpredictable when I am watching them, but they behave themselves under Fuerle's stern glare.

All economists, from every school except socialism, have observed the "spontaneous order" of the free market. It is arrogant for one school, the Austrians, to lay claim to the very concept of spontaneous order and, like Procrustes, attempt to fit their every critic into the mold of Oskar Lange.

Modern Austrians are like a boy who clears level one of his video game but then is quickly killed by the tougher monsters in level two and, afraid to repeat that bad experience, he just keeps restarting his game at the beginning until he can clear level one in record time. But, though he now goes through level one at a dead run and shoots monsters the moment they stick their heads up, the boy cannot be said to have mastered his video game because he still gets killed just as quickly as ever in level two and it is a six-level game. In the same way, the Austrians want to keep re-fighting their battle with Lange while refusing to admit that they have new enemies now – like me.[1]

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[1] A good example is Robert Murphy's recent article (QJAE, 9 (2), pp. 3-11), in which he digs up Lange's moldering remains, sticks another knife in its skeletal ribs and then re-buries it.

Fuerle prefaces the invoking of one of his laws (above) by writing, "To the extent that the conditions of the Law of Quantity Demanded apply,…" And, in general, all of his laws require that certain conditions be met before the law can be applied. This is no different than an axiomatic system like mine except that all of my theorems have the same conditions, namely the three axioms which define my theory. Fuerle's theory is also axiomatic except that each of his twenty laws have their own set of conditions. This is a weakness. It is better if all of one's results are derived from a single, unified set of assumptions, as my theorems are. It gives one's book a sense of cohesion.

Thus, I would deny Fuerle's claim that "there is no underlying law that determines the value that people place on things." In fact, there are three underlying laws, or axioms, as I call them:

1) One's value scale is totally (linearly) ordered:
   - i) Transitive; $p \leq q$ and $q \leq r$ imply $p \leq r$
   - ii) Reflexive; $p \leq p$
   - iii) Antisymetric; $p \leq q$ and $q \leq p$ imply $p = q$
   - iv) Total; $p \leq q$ or $q \leq p$

2) Marginal (diminishing) utility, u(*s*), is such that:
   - i) It is independent of first-unit demand.
   - ii) It is negative monotonic; that is, $u'(s) < 0$.
   - iii) The integral of u(*s*) from zero to infinity is finite.

3) First-unit demand conforms to proportionate effect:
   - i) Value changes each day by a proportion (called $1+\varepsilon_j$, with j denoting the day) of the previous day's value.
   - ii) In the long run, the $\varepsilon_j$'s may be considered random as they are not directly related to each other nor are they uniquely a function of value.
   - iii) The $\varepsilon_j$'s are taken from an unspecified distribution with a finite mean and a non-zero, finite variance.

If Richard Fuerle or anyone else wishes to argue that these three axioms are inapplicable to the real world, they are welcome to do so. I feel that my axioms are wisely chosen and I would be interested in hearing why they are not. However, to argue that prices are arbitrary and unpredictable and there are no underlying law that determines the value that people place on things is absurd. Such an assertion belies the existence of Fuerle's own theory, which claims to make meaningful statements about prices.

There is no direct way to counter such nihilism. Arguing with a nihilist is like lecturing a teenager, who responds to every assertion with "whatever." One can only observe that Fuerle, Mises, Keynes, this author and many others have written books about the behavior of prices. We would not have made the effort if we thought that prices were arbitrary. Hopefully, the weight of tradition will preserve the existence of economics as a legitimate field of inquiry against the attacks of the nihilists.