Geometrical Exposition of Structural Axiomatic Economics (I): Fundamentals

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Abstract

Behavioral assumptions are not solid enough to be eligible as first principles of theoretical economics. Hence all endeavors to lay the formal foundation on a new site and at a deeper level actually need no further vindication. Part (I) of the structural axiomatic analysis submits three nonbehavioral axioms as groundwork and applies them to the simplest possible case of the pure consumption economy. The graphical analysis makes the interrelations between income, profit and employment under the conditions of market clearing and budget balancing immediately evident. Part (II) applies the differentiated axiom set to the analysis of qualitative and temporal aggregation.

JEL D00, E00

Keywords new framework of concepts, structure-centric, axiom set, super-symmetry, general equilibrium, dimensionless variables, income, profit, distributed profit, retained profit, full employment

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The task of theoretical economics is to create a mental map of the whole economy without firsthand experience.

And in the social sciences it is even more obvious than in the natural sciences that we cannot see and observe our objects before we have thought about them. For most of the objects of social science, if not all of them, are abstract objects; they are theoretical constructions. (Popper, 1960, p. 135), original emphasis

That is, one has to leap from commonplace economics which trades in easy to grasp phenomena on a small scale to an extremely abstract set of foundational propositions about the economy as a whole.

Since, therefore, it is vain to hope that truth can be arrived at, either in Political Economy or in any other department of the social science, while we look at the facts in the concrete, clothed in all the complexity with which nature has surrounded them, and endeavor to elicit a general law by a process of induction from a comparison of details; there remains no other method than the à priori one, or that of “abstract speculation.” (Mill, 2004, p. 113-114)

The set of basic propositions has to reduce the vast complexity of the real thing to almost nothing. From this almost-nothingness the real world complexity then has to be logically reconstructed.

Each theory starts from a small set of foundational ‘hypotheses or axioms or postulates or assumptions or even principles’ (Schumpeter, 1994, p. 15). General equilibrium theory rests on a set of behavioral axioms (Arrow and Hahn, 1991, p. v). This approach is known to be in need of a re-design because ‘anything based on this mock-up is unlikely to fly’ (Hahn, 1981, p. 1036).

The standard set of behavioral axioms is in the present paper at first replaced by structural axioms. By choosing objective structural relationships as axioms behavioral hypotheses are not ruled out. On the contrary, the structural axiom set is open to any behavioral assumption and not restricted to the standard optimization calculus (for details see 2011b). Hence it is analytically possible to dislocate human behavior from the center of the domain to the periphery.

The methodological case for structural axiomatization has been made at length elsewhere (2012). With the basic understanding that an alternative formal foundation is reasonable as well as desirable the minimalistic structural frame is set up in Section 1. In Section 2 the axiom set is made geometrically concrete and the economic implications are elaborated. Reproducibility is ascertained with the conditions of market clearing and budget balancing. This structural supersymmetry and its implications for a move from underemployment to full employment is discussed in Section 3. In Section 4 the relation between profit and distributed profit is clarified and in Section 6 the complementarity of retained profit and saving is established. Section 7 concludes.

2
1 Axioms and definitions

The first three structural axioms relate to income, production, and expenditures in a period of arbitrary length. For the remainder of this inquiry the period length is conveniently assumed to be the calendar year. Simplicity demands that we have at first one world economy, one firm, and one product.¹

Total income of the household sector $Y$ in period $t$ is the sum of wage income, i.e. the product of wage rate $W$ and working hours $L$, and distributed profit, i.e. the product of dividend $D$ and the number of shares $N$.

$$Y = WL + DN \mid t$$ (1)

Output of the business sector $O$ is the product of productivity $R$ and working hours.

$$O = RL \mid t$$ (2)

Consumption expenditures $C$ of the household sector is the product of price $P$ and quantity bought $X$.

$$C = PX \mid t$$ (3)

The axioms represent the pure consumption economy, that is, no investment expenditures, no foreign trade, and no taxes or any other state activity. Albeit quite obvious, it is worth to emphasize that all axiomatic variables are measurable in principle. No nonempirical concepts like equilibrium, rationality or perfect competition are put into the premises.

Definitions are supplemented by connecting variables on the right-hand side of the identity sign that have already been introduced by the axioms. With (5) wage income $Y_W$ and distributed profit income $Y_D$ is defined:

$$Y_W \equiv WL \quad Y_D \equiv DN \mid t.$$ (4)

Definitions add no new content to the set of axioms but determine the logical context of concepts. New variables are introduced with new axioms.

Formal axiomatic systems must be interpreted in some domain . . . to become an empirical science. (Boylan and O’Gorman, 1995, p. 198)

The economic interpretation is rather obvious for the set of structural axioms. What deserves mention is that total income in (1) is the sum of wage income and distributed profit and not of wage income and profit. Profit and distributed profit are quite different things that have to be thoroughly kept apart.

¹ “The often heard rule that concepts are to be defined before they are used in a discussion is much too simple minded pre-Hilbertian. The only way to arrive at coherent languages is to set up axiomatic systems implicitly defining the basic concepts.” (Schmiechen, 2009, p. 344)
2 An open opening

Figure 1 makes the axioms immediately concrete. The diagram looks like the familiar Cartesian coordinates. However, since there is no use for negative values the four axes represent the positive values of the variables employment $L$, income $Y$, consumption expenditures $C$, quantity bought $X$ and output $O$, respectively. The bisecting line in the northwestern quadrant mirrors income from the horizontal to the vertical axis. This facilitates the direct comparison of the nominal values of income and consumption expenditures. The quadrants are numbered according to the axioms they enclose.

![Diagram](image)

**Figure 1:** At first the product market is not cleared and the household sector’s budget is not balanced

In the 1st quadrant wage income $Y_W$ is given as product of wage rate $W$ and working hours $L$. The wage rate is equal to the tangent function of the angle $\alpha$ at $L = 1$. Angles are not needed for our analysis hence they are denoted with the respective economic variables that are used for graphical multiplication. Total income $Y$ consists at the beginning only of wage income $Y_W$. Distributed profit $Y_D$ as constituent of the 1st axiom (1) has been set to zero and shall be considered separately in Section 5.

In the 2nd quadrant output $O$ is given as product of productivity $R$ and working hours $L$. The productivity is determined by the underlying production process and may vary with labor input. The 2nd axiom (2) should therefore not be interpreted as a linear production function. It is compatible with increasing, constant and decreasing returns. Employment and productivity changes can always be treated separately and then combined; it is much like vector decomposition and addition.
Since we are not 
_a priori_ wedded to the marginal principle there is no need to
confine the analysis to decreasing returns. It should be noted in passing that any
approach that operates solely with decreasing returns cannot claim to be general.

In the 3nd quadrant consumption expenditures \( C \) is given as product of price \( P \)
and quantity bought \( X \).

Since the quantity produced \( O \) is here larger than the quantity bought \( X \) the firm
that at the moment represents the entire business sector has an unsold quantity \( \Delta O \)
left over at period end which has to be taken into stocks. The change of inventory in
period \( t \) is defined as:

\[
\Delta O \equiv O - X \mid t. \tag{5}
\]

In the period under consideration the product market is not cleared.

Consumption expenditures \( C \) in the 1st quadrant is less than income \( Y \), that is,
the households save. Financial saving is defined as:

\[
\Delta S_{fi} \equiv Y - C \quad \text{here} \quad Y = Y_W \mid t. \tag{6}
\]

In the period under consideration the household sector’s budget is not balanced.

Is this configuration realistic? Of course. The households can save whenever and
whatever they want and it is a normal incident that a firm takes part of current output
into stocks. All depends on what happens in the subsequent periods. If the firm sells
the quantity \( \Delta O \) in the next period in addition to the current output the inventory
vanishes again. Hence, seen over two periods, the product market is cleared (see
Part (II)). Problems arise, though, if the stock of unsold products accumulates over
a longer time span, that is, if the configuration of Figure 1 is identically reproduced
in subsequent periods. It is similar with saving. If the households dissave in the
next period their budget is balanced over two periods.

Problems arise in the pure consumption economy if the households stubbornly
continue saving because this drives firms to the brink of bankruptcy. The business
sector’s financial profit in period \( t \) is defined with (7) as the difference between the
sales revenues – for the economy as a whole identical with consumption expenditures
\( C \) – and costs – here identical with wage income \( Y_W \):^3

\[
\Delta Q_{fi} \equiv C - Y_W \mid t. \tag{7}
\]

Since distributed profit in (1) is zero we have in this simple case \( Y = Y_W \). Hence
from (7) in combination with (6) follows:

\[
\Delta Q_{fi} \equiv C - Y \quad \Rightarrow \quad \Delta Q_{fi} \equiv -\Delta S_{fi} \mid t. \tag{8}
\]

The business sector makes a financial loss in the period under consideration
which is equal to the financial saving of the household sector. This is certainly not a

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^2 For the treatment of nonfinancial saving see (2011a, Sec. 4.2).

^3 Profits from changes in the value of nonfinancial assets are neglected here. For details about effects
of changes of inventory on profits see (2011a).
healthy situation over a longer time span. What is worse, the business sector cannot
do much to change the situation because the households are perfectly free in their
decision to spend or to save their income in the current period.

The definition of financial profit as such does, of course, not explain how
profit comes about, it only specifies explicitly what one is talking about. Since the
definition of an apple does not explain how Nature produces apples we shall in the
following determine the overall conditions that enable the firms to make a profit,
that is, to answer the question how it comes about that revenues could be greater
than wage costs. This is an old chestnut of political economy which has never been

There is, to be sure, not much use in speculating about what the households
or our single firm would or could do in the given situation or to introduce some
convenient behavioral assumptions that make that household saving is always zero
in the pure consumption economy. What can be done, though, is to determine the
structural conditions for a reproducible period configuration.

3 Reproducibility and employment indifference

After the implementation of the conditions of market clearing $X = O$ and budget
balancing $C = Y$ (at the moment $Y = Y_w$) the pure consumption economy looks
as shown in Figure 2. This configuration is referred to as supersymmetric. It is
important to notice that the two conditions are not a constituent part of the axiom
set but an – in principle – arbitrary addendum.

![Figure 2: Implementation of the conditions of product market clearing and budget balancing](image-url)
The market clearing price then follows from (1) and (3) as:

\[ P = \frac{W}{R} \]

if \( X = O; \) \( C = Y \) here \( Y = Y_W \) \( \text{if} \).  

(9)

The market clearing price is equal to unit wage costs. Hence profit per unit of output is zero and therefore overall profit is zero. This configuration is reproducible. The firm sells its period output completely and fully recoups its wage costs. It is worth emphasizing that the market clearing price is unequivocally determined by the three axioms and the supersymmetry conditions. It is therefore impossible to add independent demand and supply functions. There is simply no formal room left for additional behavioral assumptions or some occult market forces that equalize price and unit wage costs.

The market clearing price in (9) is independent of employment. Hence, if employment \( L \) changes while wage rate \( W \) and productivity \( R \) remain unaltered then the price \( P \) remains constant. This case is depicted in Figure 3.

![Figure 3: Different employment levels under the condition of market clearing and budget balancing with constant productivity and wage rate](image)

It is therefore possible that the economy moves from underemployment \( L_u \) to full employment \( L_f \) without any change of wage rate and price if the productivity is given. If productivity and wage rate change on the move to full employment this affects only the market clearing price as given by (9). Since financial profit is zero under the condition of balanced budget it is of no consequence for the business sector whether the economy operates at full employment or underemployment.
Profit is zero in both cases. Business could therefore be indifferent about various employment levels. A wage rate reduction is no precondition for attaining full employment, it would only lower the market clearing price. From (9) follows immediately for the real wage:

\[
\frac{W}{P} = R \mid t. \tag{10}
\]

Whatever happens to the wage rate is of no consequence for the real wage which is invariably equal to the productivity that, in turn, is objectively determined by the production process. Under the condition of increasing returns the move from underemployment to full employment entails an increasing real wage. With a constant productivity in the relevant range the real wage does not change at all. Under this condition the move to full employment is indifferent for the already employed and beneficial for the hitherto unemployed.

Under the conditions of product market clearing and budget balancing a move from underemployment to full employment that is perfectly indifferent for both the already employed wage earners and the business sector presupposes a constant productivity. Wage rate changes in either direction are immaterial. If the business sector is not indifferent and does not move to a higher employment level unless profit is greater than zero then full employment is unattainable, that is, the business sector prevents a Pareto-optimal employment expansion. The supersymmetric consumption economy is reproducible on any employment level.

4 The logical emergence of profit

Profit has been defined with (7). In explicit form, after the substitution of (3) and (4), this definition is identical with that of the theory of the firm:

\[
\Delta Q_{fi} \equiv PX - WL \mid t. \tag{11}
\]

Using the first axiom (1) one gets alternatively:

\[
\Delta Q_{fi} \equiv C - Y + Y_D \mid t. \tag{12}
\]

The three definitions are formally equivalent. Profit can be seen from different perspectives. Taken together, the three perspectives make a comprehensive view. If distributed profit \(Y_D\) is set to zero in (12), then profit or loss of the business sector is determined solely by consumption expenditures and wage income. For the business sector as a whole to make a profit consumption expenditures \(C\) have in the simplest case to be greater than wage income \(Y_W\) as shown in Figure 4.

The price \(P\) is determined by the axioms (1) to (3) and the condition of market clearing:

\[
P = \frac{C}{RL} \quad \text{if} \quad X = O \mid t. \tag{13}
\]
We define the expenditure ratio $\rho_E$ as:

$$\rho_E \equiv \frac{C}{Y} \mid t. \quad (14)$$

Together with (13) this yields the market clearing price in the general form:

$$P = \rho_E \frac{W}{R} \quad \text{if} \quad X = O \mid t. \quad (15)$$

The market clearing price is higher or lower than unit wage costs depending on the expenditure ratio $\rho_E$. In Figure 4 the profit per unit is positive because $\rho_E > 1$.

So that profit comes into existence for the first time in the pure consumption economy the household sector must run a deficit at least in one period. As long as the households spend their wage incomes fully the business sector will not make a loss but it will not see any profits either.

The logical explanation of profit therefore consists in: revenues can only be greater than costs if, in the simplest of all possible cases, consumption expenditures are greater than wage income. The existence of profit for the economy as a whole does neither depend on the working hours, the wage rate nor on productivity. Variations of these variables are compensated for by the market clearing price.

Some economists start from the seemingly innocuous presumption that the value of output is equal to the value of incomes. Figure 4 makes it immediately clear that this presumption is unfortunate because it holds only in the special case of budget balancing. To start with this assumption therefore amounts to an analytical self-
lock-in. There is nothing in the real world that makes the equal-value presumption come true. The general case is $\rho_E \neq 1$.

The household sector’s initial deficit in turn makes the inclusion of the financial sector mandatory. A theory that does not include at least one bank that supports the concomitant credit expansion cannot capture the essential features of the market economy (for the inclusion of money and banking see 2011c; 2011d).

Mention should be made that neither neoclassicals nor Keynesians ever came to grips with profit (Desai, 2008), (Tómasson and Bezemer, 2010).

5 Distributed profit and profit

Once profit has come into existence for the first time (that is: logically – a historical account is a quite different matter) the business sector has the option to distribute or to retain it. This in turn has an effect of profit. This effect is captured by (12) but it is invisible in (11). Both equations, though, are formally equivalent. Profit distribution and the spending out of distributed profit is depicted in Figure 5.

![Figure 5: Distributed profit and spending out of distributed profit under the condition of market clearing and budget balancing](image)

If the household sector’s budget is balanced, i.e. if consumption expenditures are equal to total income in (12) then profit is, as a corollary, equal to distributed profit:

$$\Delta Q_{fi} \equiv C - Y + Y_D \quad \Rightarrow \quad \Delta Q_{fi} = Y_D \quad \text{if} \quad C = Y \quad | t. \quad (16)$$
The market clearing price follows from the axioms (1) to (3) and is given by:

\[ P = \frac{W}{R} + \frac{Y_D}{RL} \quad \text{if} \quad X = O; \quad C = Y |t. \]  

(17)

The market clearing price is higher than unit wage costs in the supersymmetric case if distributed profit is greater than zero. Given the amount of distributed profit as well as wage rate and productivity the price varies with employment. With increasing employment the market clearing price falls.

The determinants of profit look essentially different depending on the perspective. For the firm price \( P \), quantity \( X \), wage rate \( W \), and employment \( L \) in (11) appear to be all important for profit; under the broader perspective of (12) these variables play no independent role. The profit definition provokes a cognitive dissonance between the micro and the macro view. It is therefore worthwhile to realize that equations (7), (11), (12) are not only equivalent but indeed indispensable for a consistent view of profit.

Profit \( \Delta Q_f \) and distributed profit \( Y_D \) is clearly distinguishable. The latter is a flow of income from the business to the household sector analogous to wage income. By contrast, profit is the difference of flows within the business sector. Profit is not connected to a factor input. So far, we have labor input as the sole factor of production and wage income as the corresponding factor remuneration. Since the factor capital is nonexistent in the pure consumption economy, profit cannot be assigned to it in functional terms. And since profit cannot be counted as factor income there is no place for it in the theory of income distribution.

The individual firm is blind to the structural relationship that is given by (12). On the firm’s level profit is therefore subjectively interpreted as a reward for innovation or superior management skills or higher efficiency or toughness on wages or for risk taking or capitalizing on market imperfections or as the result of monopolistic practices. These factors play a role when it comes to the distribution of profits between firms and these phenomena become visible when similar firms of an industry are compared. Because of this, it is not wise to take the considerations of the individual firm’s management as analytical starting-point and then to generalize. The microeconomic approach is inherently prone to the fallacy of composition.

Under the condition \( C = Y \) financial profit \( \Delta Q_f \) is according to (16) numerically equal to distributed profit \( Y_D \) as in Figure 5. The fundamental difference between the two variables does not catch the eye in this limiting case. The equality of profit and distributed profit is an implicit feature of equilibrium models. These have no counterpart in reality. In the real world holds \( C \neq Y \), therefore profit and distributed profit are never equal. Models that are based on the familiar definition total income \( \equiv \text{wages + profits} \) are erroneous because profit and distributed profit is not the same thing.
6 Retained profit and saving

Profits can either be distributed or retained. If nothing is distributed, then profit adds entirely to the financial wealth of the firm. Retained profit $\Delta Q_{re}$ is defined for the business sector as a whole as the difference between profit and distributed profit in period $t$:

$$\Delta Q_{re} \equiv \Delta Q_{fi} - Y_D \Rightarrow \Delta Q_{re} \equiv C - Y \mid t. \quad (18)$$

Retained profit is due to (12) equal to the difference of consumption expenditures and total income.

Financial saving is given by (6) as the difference of income and consumption expenditures. In combination with (18) follows:

$$\Delta Q_{re} \equiv -\Delta S_{fi} \mid t. \quad (19)$$

Financial saving and retained profit always move in opposite directions. Let us call this the complementarity corollary because it follows directly from the definitions themselves. The corollary asserts that the complementary notion to saving is not investment but negative retained profit. Positive retained profit is the complementary of dissaving. Since there is neither capital nor inventory investment in the pure consumption economy with market clearing the familiar IS-equilibrium evidently cannot hold. This crucial point, though, is worth its own analysis (for details see 2011e).
7 Conclusion

If we ask, ‘What is the most adequate model of behaviour for economics?’ we implicitly assume that economics actually needs a model of behaviour; hence, we already assume psychologism of a kind. (Hudík, 2011, p. 147)

Behavioral assumptions, rational or otherwise, are not solid enough to be eligible as first principles of theoretical economics. Neither are they needed. The present paper excludes psychologism and suggests three nonbehavioral axioms as groundwork for the consistent reconstruction of the elementary consumption economy. The main results of the rather straightforward geometrical analysis are:

- The consumption economy that is at first given with the bare set of structural axioms is not reproducible over a longer time span. This is the original and general case.

- The addition of the conditions of market clearing and budget balancing to the axiom set yields a reproducible consumption economy with indifferent employment levels for the business sector.

- The market clearing price is unequivocally determined by the axiom set and the supersymmetry conditions. It is therefore impossible to add independent demand and supply functions. There is no formal room left for behavioral assumptions.

- Under the condition of supersymmetry wage rate changes in any direction are of no consequence for the move from underemployment to full employment.

- The logical explanation of profit consists in: revenues can only be greater than costs if, in the simplest of all possible cases, consumption expenditures are greater than wage income.

- Under the condition of budget balancing profit is numerically equal to distributed profit. In the real world the households sector’s budget is not balanced, therefore profit and distributed profit are never equal.

- Models that are based on the familiar definition total income \(\equiv\) wages + profits are erroneous because profit and distributed profit is not the same thing.

- The complementary notion to saving is not investment but negative retained profit. Positive retained profit is the complementary of dissaving.
References


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